Design of domes.

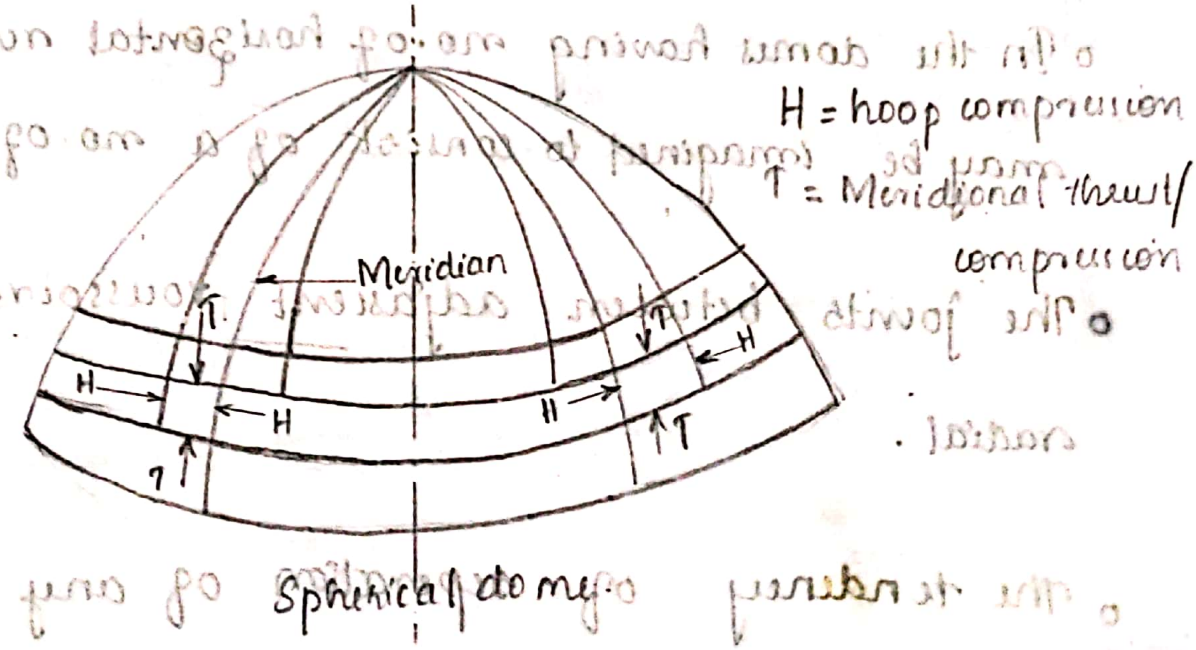
- The roof and floor of a circular tank may be designed as a dome.
- A dome is a type of shell obtained by the revolution of a regular curve like parabolic, circular or elliptical curve about one of its axis.
- A dome is economical as compared to a slab as it carries the load through membrane action resulting in negligible bending moment and

shear forces.

- A dome may be analysed by assuming it to consist of a no. of horizontal rings placed one over the other with gradually reducing diameters towards top.
- The equilibrium of each ring is considered as independent of rings placed above it.
- Any section of the ring is subjected to following types of forces

1. Hoop or circumferential tension or compression
2. Meridional thrust.

- The hoop tension or compression acts along the circumference of the dome and the meridional thrust acts along the meridian direction.



Meridional thrust.

- In a dome, the successive horizontal rings subtend equal angle at the center of the sphere.
- The joints between successive horizontal rings is radial.
- Every horizontal ring supports the load of the rings above it, and transmits it to the one below it.
- The reaction between the rings is tangential to the curved surface, giving rise to compression along the meridian.
- This compressive stress is called meridional thrust or meridional compression.

Hoop or circumferential tension / compression.

- In the domes having no. of horizontal rings, which may be imagined to consist of a no. of voussoirs.
- The joints between adjacent voussoirs of ~~a~~ the ring ~~ouwing~~ are radial.
- The tendency of seperation of any voussoiors

will be prevented because of its wedge shape, and therefore hoop compression will be caused in each ring (θ value less than $51°49'$)

For ~~sp~~ spherical domes,

meridional thrust $T = \frac{wR}{1+\cos\theta} + \frac{W}{2\pi R(\sin^2\theta)^2}$

Hoop stress, $f = \frac{wR}{t}\left[\cos\theta - \frac{1}{1+\cos\theta}\right] - \frac{W}{2\pi R t \sin^2\theta}$

R = radius of the dome.

w = uniformely distributed load acting on the dome.

W = point load (any specific dead load) acting on the dome.

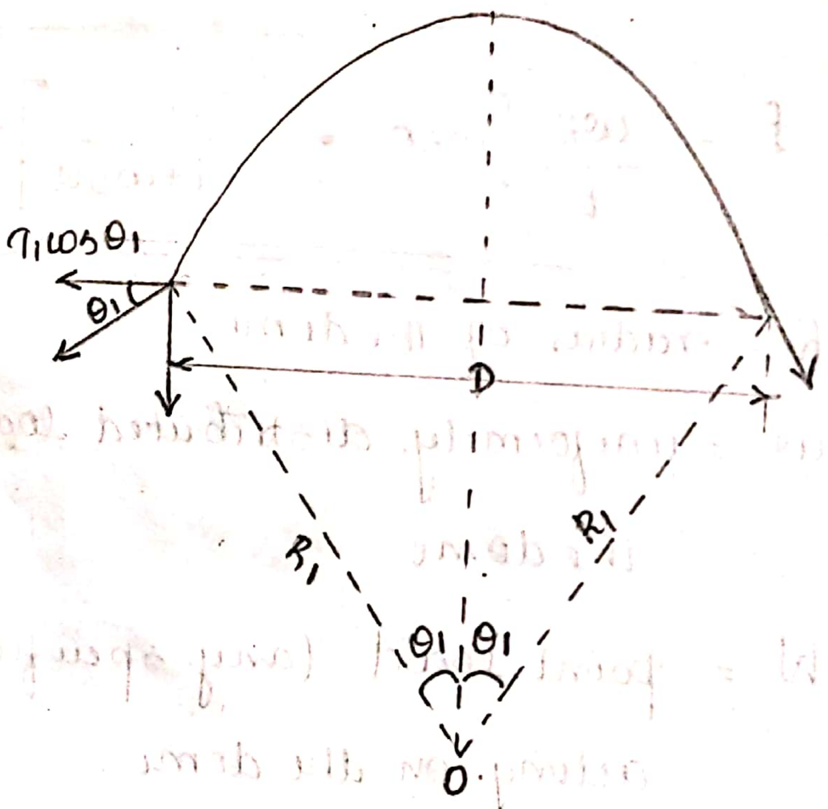
θ = Vertical angle at the section measured from vertical axis of the dome.

- ~~The value of hoop~~ compression ~~can be found by following equation.~~
- The θ value more than $51°49'$, hoop tension will develop and for θ less than $51°49'$, hoop compression

will develop.

Ring beam

- The ring beam subjected to hoop tension caused due to the horizontal component of the meridianal thrust developed at the base of the dome.



- Hoop tension or compression on the ring beam $(T_R) = T_1 \times \cos\theta \times \frac{D}{2}$

where, D = Diameter of the dome.

$$T_1 = \frac{w R_1}{1+\cos\theta_1} + \frac{W}{2\pi R_1 \sin^2\theta_1}$$

0 The area of steel required to take this hoop tension is calculated as $\frac{T_R}{\sigma_{st}}$

0 The area of ring beam required to take this hoop tension is calculated by limiting the hoop stress developed to be within permissible limit.

i.e. $$\frac{T_R}{A+(m-1)A_{st}} \le \sigma_{ct}$$

where,

A = Area of the ring beam

m = modular ratio

A_{st} = Area of steel provided in ring beam

σ_{ct} = permissible direct tensile stress in concrete.

7 ~~Design~~. A spherical dome of a water tank of span 6m has a rise of 1.2m. It carries all inclusive distributed loads of ~~600N/m²~~ 6000 N/m² and a lantern load of ~~800N~~ 8000N at the crown. Design the dome. Use M20 concrete and Fe415 steel.

Given data,

$w = 6000 N/m^2$

$W = 8000 N$.

(wt of lantern)

Span = 6m.

Rise = 1.2m

Grade of concrete = M_{20}

Grade of steel = Fe415

σ_{ct} = 1.[illegible] N/mm^2

σ_{st} = 150 N/mm^2

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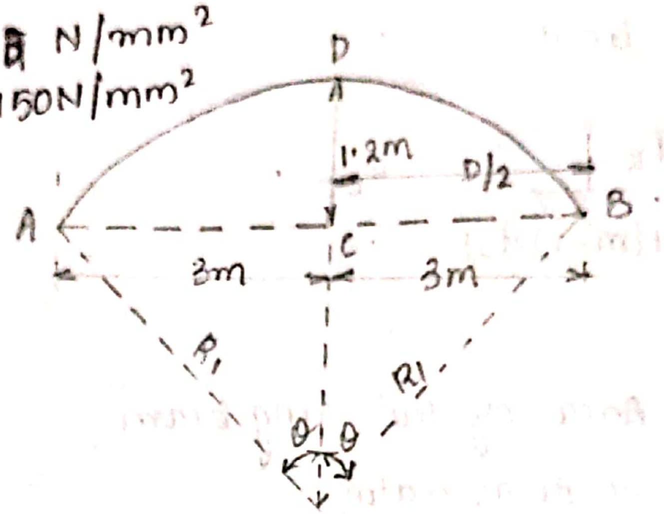


Let us use the property of circle

$$(2R_1 - CD)\,CD = \left(\frac{D}{2}\right)\left(\frac{D}{2}\right) = \left(\frac{D}{2}\right)^2$$

Find the value of radius of dome by using the property of circle

$$(2R_1 - 1.2)\,1.2 = 3^2$$

$$2R_1 - 1.2 = 7.5$$

$$2R_1 = 8.7$$

$$R_1 = \underline{\underline{4.35\ m}}.$$

$\sin\theta = \frac{3}{R_1}$

$\theta = \sin^{-1}\left(\frac{3}{4.35}\right)$

$\theta = 43^\circ 36'$ $< 51^\circ 49'$ so hoop compression occurs

minimum steel is to be provided

$Ast = \frac{0.3 \times c/s}{100}$

And if $\theta > 51^\circ 49'$, hoop tension occurs

And steel is to be designed

$\sin\theta = 0.6896$

$\cos\theta = 0.724$

~~Med~~ Meridional thrust per m run at springing level

$$T = \frac{wR}{1+\cos\theta} + \frac{W}{2\pi R (\sin\theta)^2}$$

$$= \frac{6000 \times 4.35}{1 + 0.724} + \frac{8000}{2\pi \times 4.35 \times (0.6896)^2}$$

$T = 15753.49\ N.$

Let us assume the thickness of dome is 100mm

Compressive stress due to meridional thrust $= \frac{Load}{area}$

$$= \frac{15753.49}{1000 \times 100}$$

$$= 0.157\ N/mm^2$$

$$\text{Hoop stress } (f) = \frac{wR}{t}\left[\cos\theta - \frac{1}{1+\cos\theta}\right] - \frac{W}{2\pi R t \sin^2\theta}$$

$$= \frac{6000 \times 4.35}{100 \times 10^{-3}}\left[0.724 - \frac{1}{1+0.724}\right] - \frac{8000}{2\pi \times 4.35 \times 100 \times 10^{-3} \times \sin^2 (0.689)}$$

$$= 31416.9124\ N/m^2$$

$= \underline{0.314 \ N/mm^2}$

As per IS 456 page no: 81, table 21

the permissible direct compressive stress for M_{20} concrete is

$\underline{5 N/mm^2}$

The hoop stress and stress due to meridional thrust are very low. Hence a minimum of 0.3% steel will be provided in each principle direction.

$$\text{Minimum steel requirement} = \frac{0.3}{100} \times 1000 \times 100 \quad (b \times d)$$

$$= \underline{300 mm^2}$$

$$\text{Spacing of 10mm } \phi \text{ bars} = \frac{\frac{\pi}{4} \times (10)^2 \times 1000}{300}$$

$$= \underline{261.799 mm^2}$$

$$\simeq \underline{260 mm}.$$

Ring beam.

Horizontal component of meridional thrust $= T.\cos\theta$.

$$= 15753.49 \times 0.724$$

$$= \underline{11405.526 N}.$$

Hoop tension in the ring beam (T_R) = $T_{c050} \times D/2$

$= 11405.526 \times 3$

$= \underline{34216.58 \text{ N}}$

Safe stress in steel σ_{st} = 150 N/mm^2

$$A_{st} = \frac{T_R}{\sigma_{st}} = \frac{34216.58}{150} = \underline{228.11 \text{ mm}^2}$$

Provide 4 bars of 12mm diameter.

$$A_{st\,pro} = 4 \times \frac{\pi}{4} \times 12^2 = \underline{452.389 \text{ mm}^2}$$

$$m = \frac{280}{3\sigma_{cbc}} = \frac{280}{3 \times 6.66} = \underline{14}$$

$$\frac{T_R}{A + (m-1)A_{st}} \leq \sigma_{ct}$$

$$\frac{34216.58}{A + (14-1)452.389} \leq 1.2$$

$$34216.58 = 1.2A + 7057.2684$$

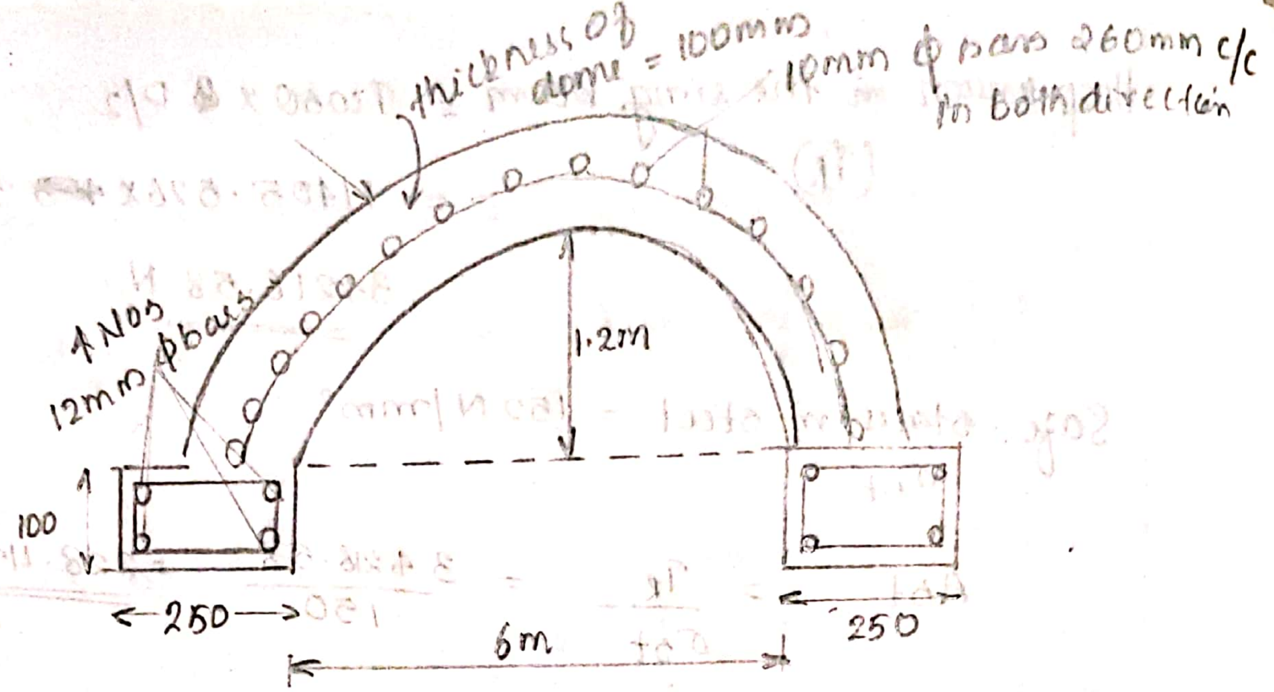
$$A = \underline{22632.75 \text{ mm}^2}$$

Assume width of the beam = 250mm

$$\text{Depth of beam} = \frac{22632.75}{250} = \underline{90.53 \text{ mm}}$$

$\approx \underline{100 \text{mm}}$

Cross section of ring beam = 250x100mm



8 Design a spherical dome for a circular water tank of 500 kl capacity. The tank is supported on a masonry tower. The depth of water in tank is 5m. Rise of the dome is 2m. Take live load on the dome as $1 kN/m^2$

$\frac{\pi}{4} \times D^2 = \frac{capacity}{depth\ of\ water}$

⇒ Given,

capacity of tank = 500 kl = 500 m^3

H = 5m

$\frac{\pi}{4} \times D^2 \times h = 500$

$\frac{\pi}{4} \times D^2 \times 5 = 500$

$\therefore D = 11.28 \approx \underline{11.3m}$

By using the property of circle.

$(2R_1 - Rise) Rise = \left(\frac{D}{2}\right)^2$

$$(2\times R_1 - 2)\,2 = \left(\frac{11.3}{2}\right)^2$$

$$(2R_1-2)\,2 = 31.92$$

$$2R_1 - 2 = 15.96$$

$$\therefore R_1 = 8.98\,m$$

$$\sin\theta = \frac{11.3/2}{R_1}$$

$$\therefore \theta = \sin^{-1}\left[\frac{11.3/2}{8.98}\right]$$

$$= 38^\circ 59'$$

$38^\circ 59' < 51^\circ 49'$ so hoop compression will be occur.

$$\sin\theta = 0.6290$$

$$\cos\theta = 0.777$$

Meridional thrust per 'm' run at springing wall

$$T = \frac{wR}{1+\cos\theta}$$

$$= \frac{1000\times 8.98}{1+0.777}$$

$$= 5053.46\,N$$

Let us assume the thickness of dome is 100mm

comp. stress due to meridional thrust = $\frac{\text{Load}}{\text{area}}$

$$= \frac{5053.46}{1000\times 100}$$

$$= 0.0505\,N/mm^2$$

$$\text{Hoop stress} = \frac{wR}{t}\left[\cos\theta - \frac{1}{1+\cos\theta}\right]$$

$$= \frac{1000 \times 9.98}{100 \times 10^{-3}}\left[0.777 - \frac{1}{1+0.777}\right]$$

$$= 19263.63 \ N/m^2$$

$$= \underline{0.0192 \ N/mm^2}$$

As per IS 456 pg no 81, table 21.

Assume M20 concrete and Fe 415 steel.

The permissible direct compressive stress for concrete is 5 N/mm2.

$$\text{Min steel requirement} = \frac{0.3}{100} \times 1000 \times 100$$

$$= \underline{300 \ mm^2}$$

$$\text{Spacing of 10mm } \phi \text{ bars} = \frac{\frac{\pi}{4}(10^2) \times 1000}{300}$$

$$= 261.79 \ mm$$

$$\approx \underline{260 \ mm}$$

→ Ring Beam

Horizontal component of meridional thrust

$$= T\cos\theta$$

$$= 5053.46 \times 0.777$$

$$= 3926.53 \ N$$

Hoop tension in the ring beam

$= T_1 \cos\theta \times D/2.$

$= 3926.53 \times 0.777 \times \frac{11.3}{2}$

$= 17244.014N.$

safe stress in steel $= 150 N/mm^2$.

$A_{st} = \frac{T}{\sigma_{st}} = \frac{17244.04}{150}$

$= 114.96 mm^2$

Provide 4 bars of 12mm ϕ.

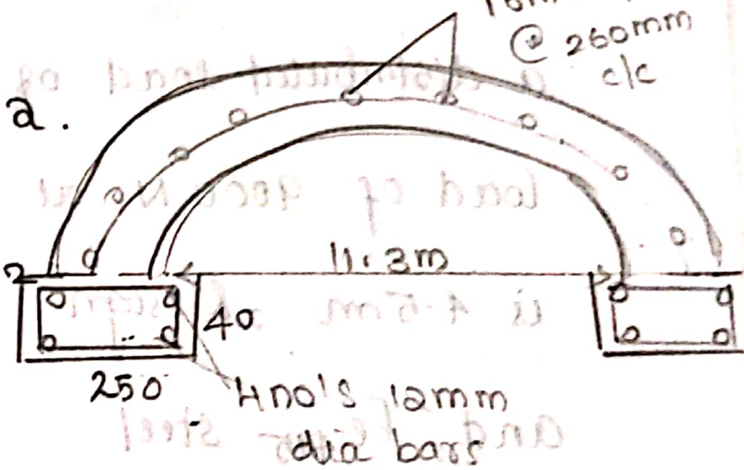
$A_{st\,prov} = 4 \times \frac{\pi}{4} \times 12^2 = 452.33 mm^2$

$452.33 > 114.96 mm^2$

$\frac{T_R}{A + (m-1)A_{st}} \le \sigma_{ct}$

$\frac{17244.014}{A + (14-1)452.33} = 1.2.$

$\therefore A = 8490.11 mm^2$

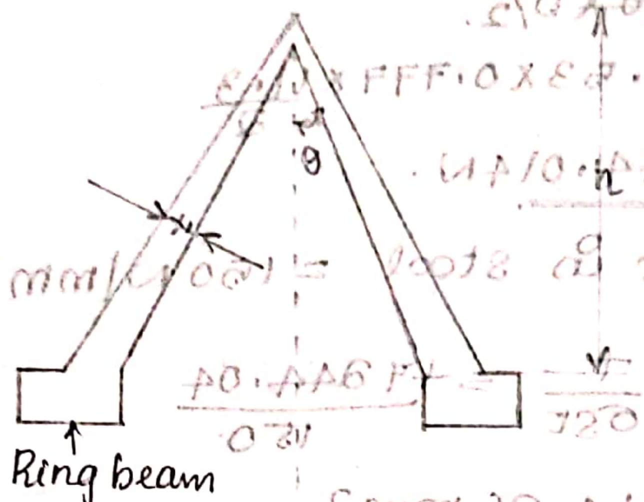


Width of beam = 250mm

Depth of beam = 33.96 ≃ 40mm.

c/s of ring beam = 250mm x 40mm.

Design of conical domes:



Meridional thrust per m run $= \frac{wh \sec^2\theta}{2} + \frac{W \sec\theta}{2\pi h \tan\theta}$.

Hoop stress $= \frac{wh}{t} \tan^2\theta$. (here point load is not considered / neglected)

9 A conical dome has a base diameter of 9m. It carries a distributed load of 4900 N/m² and a concentrated load of 9000 N at the vertex. The height of the dome is 4.5m. Design the dome. Use M20 concrete and Fe_{415} steel.

Solution

$w = 4900 N/m^2$

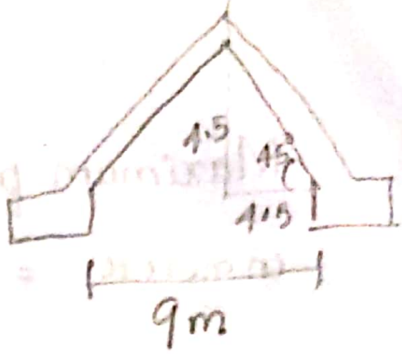
$W = 9000 N$

Height of dome = 4.5m.

Grade of concrete = ~~20N~~ M_{20}

Grade of steel = Fe_{415}.

Diameter of dome = 9m.



$\sigma_{cbc} = \frac{20}{3} = 6.66 \simeq 7 \text{ N/mm}^2$

$\sigma_{ct} = 1.2 \text{ N/mm}^2$

$\sigma_{st} = 150 \text{ N/mm}^2$

$\tan\theta = \frac{4.5}{4.5}$

$\theta = 45^\circ$

<45 deg

Assume a thickness of 100mm for the dome slab.

The meridional thrust per m run at the base

↓

compressive force; $T = \frac{wh}{2} \sec^2\theta + \frac{W\sec\theta}{2\pi h \tan\theta}$.

$T = \frac{4900 \times 4.5}{2} \sec^2 45 + \frac{9000 \sec 45}{2\pi \times 4.5 \tan 45}$

$= 22050 + 450.15$

$= 22500.158 \text{ N}$.

∴ Compressive stress due to meridional thrust $= \frac{T}{\text{Area}}$

$= \frac{22500.158}{1000 \times 100}$

$= 0.225 \text{ N/mm}^2$

✗ Only udl is considered as pt load act along the center line its effect is neglected and pt load is not considered in hoop stress

↑

Hoop stress $= \frac{wh \tan^2\theta}{t \times 1}$ (area) $= \frac{4900 \times 4.5 \times \tan^2 45}{100 \times 10^{-3}}$

$= 220500 \text{ N/m}^2$

$= 0.2205\ N/mm^2$

Maximum permissible direct compressive stress of M_{20} concrete $= 5\ N/mm^2$

The hoop stress and stress due to meridional thrust are very low. Hence a minimum of 0.3% steel will be provided

Minimum steel required $= \frac{0.3}{100} \times 1000 \times 100$

$= 300 mm^2$

Using 8mm ϕ bars,

$$\text{Spacing} = \frac{\frac{\pi}{4} \times (8)^2 \times 1000}{300} = 167.55 mm$$

$\simeq 165 mm$

Hence provide 8mm bars @ 165mm c/c along the meridian and latitudes.

Ring beam.

Horizontal component of meridian thrust at the base $= T\cos\theta$.

$= 22500 \cos 45 = 15909.9\ N$

Hoop tension in the ring beam $T_R = \frac{Q}{} \; T\cos 0 \times \frac{D}{2}$

$= 15909.9 \times 4.5$

$= 71594.56\ N$.

Safe Allowing a stress of $150\ N/mm^2$ in the reinforcement

$A_{st} = \frac{T_R}{\sigma_{st}} = \frac{71594.56}{150} = 477.297\ mm^2$.

Providing ~~4~~ 2 nos ~~16~~ 16mm and 2nos 12mm diameter bars.

A_{st} provided $=$ ~~$4 \times \frac{\pi}{4} \times (12)^2 + 2 \times 452.38\ mm^2$~~

$= 2 \times \frac{\pi}{4} \times (12)^2 + 2 \times \frac{\pi}{4} \times (16)^2 = 628.31\ mm^2$

$A_{st\,req} > A_{st\,pr}$

$\therefore A_{st}$

$m = \frac{280}{3\sigma_{cbc}}$

$\sigma_{ct} \geq \frac{T_R}{A + (m-1)A_{st}}$

$= \frac{280}{3 \times \frac{20}{3}}$

$= 14$

$\frac{71594.56}{A + (14-1)\ \text{~~477.297~~}\ 628.318} \leq 1.2$

$71594.56 = 1.2A + 9801.769$

$A = 51493.99\ mm^2$

Providing of width of 250mm

∴ Depth of ring beam $= \frac{51493.99}{250}$

$= 205.91\,mm \approx 210\,mm$